UNIVERSITY OF CAMBRIDGE

ENGINEERING ADMISSIONS ASSESSMENT **D564/11**

2022 **60 minutes**

SECTION 1

INSTRUCTIONS TO CANDIDATES

0725625112

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A Mathematics and Physics (20 questions)
Part B Advanced Mathematics and Advanced Physics (20 questions)

You are **strongly** advised to divide your time equally between the two parts: 30 minutes on **Part A** and 30 minutes on **Part B**. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 35 printed pages and 5 blank pages.

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PART A Mathematics and Physics

1 Which one of the following is a simplification of

$$y\left(\frac{3x^{\frac{1}{2}}z}{y^{3}}\right)^{2} = y\left(\frac{9xz^{2}}{y^{6}}\right) = \frac{9xz^{2}}{y^{5}}$$

A $\frac{3xz^{2}}{y^{4}}$

B $\frac{3xz^{2}}{y^{5}}$

C $\frac{9x^{\frac{1}{2}}z^{2}}{y^{5}}$

D $\frac{9xz^{2}}{y^{4}}$

E $\frac{9xz^{2}}{y^{5}}$

F $\frac{9x^{\frac{5}{2}}z^{2}}{y^{5}}$

2 There is a constant current in a conducting wire. A charge of 20 C passes through the wire in 1.5 minutes.

An 18 cm straight section of this wire lies in a uniform magnetic field. This section of wire is perpendicular to the direction of the field. The magnetic field strength is 0.15 T.

What is the magnitude of the magnetic force on this section of wire?

A 0.0060 N

B 0.36 N

C 0.60 N

D 0.81 N

E 36 N

F 49 N

G 81 N

H 4900 N

$F = BIL$ (since direction of current and $B \perp$)

$$I = \frac{\Delta Q}{\Delta t} = \frac{20}{1.5 \times 60} = \frac{20}{90} = \frac{2}{9}\text{ A}$$

$$F = BIL = 0.15 \times \frac{2}{9} \times 0.18$$

$$= (0.15 \times 2)\left(\frac{0.18}{9}\right)$$

$$= 0.3 \times 0.02 = 0.006\text{ N}$$

3 Find the complete set of values of x that satisfy the inequality

$$\frac{3}{4}(5-x)-\frac{1}{2}(6-x)-x < 0$$

A $x<\frac{1}{3}$

B $x>\frac{1}{3}$

C $x<\frac{3}{5}$

D $x>\frac{3}{5}$

E $x<\frac{3}{4}$

F $x>\frac{3}{4}$

G $x<\frac{3}{2}$

H $x>\frac{3}{2}$

$$\frac{15}{4}-\frac{6}{2}-\frac{3}{4}x+\frac{1}{2}x-x<0$$

$$\frac{15-12}{4}+\frac{-3+2-4}{4}x<0$$

$$\frac{3}{4}-\frac{5}{4}x<0$$

$$3-5x<0$$

$$x>\frac{3}{5}$$

4 Two identical resistors are connected in parallel to a 6.0 V battery. The two resistors dissipate a total power of 0.15 W.

One of these resistors is removed from the circuit and connected to a 12 V battery.

How much charge passes through this resistor in 6.0 minutes?

A 0.025 C

B 0.050 C

C 0.15 C

D 0.30 C

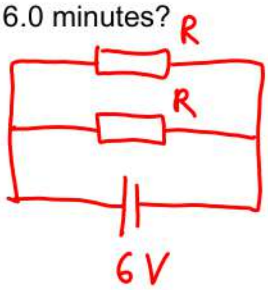
E 0.75 C

F 1.5 C

G 9.0 C

H 18 C

1.



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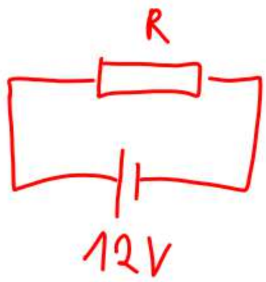


$P_{TOT} = 0.15\,W$

$P = 2U \times \frac{U}{R} = \frac{2U^2}{R}$

$R = \frac{2U^2}{P} = \frac{2\times 36}{0.15}$

$I = ?$ $Q = ?$

$Q = It$

$= \frac{U}{R}t$

$= \frac{12\times 0.15}{2\times 36}\times 6\times 60$

$\frac{12\times 0.15\times 10}{2} = 6\times 1.5$

$= 9C$

5 Rob keeps a record of what he earns each day.

On Monday, he earned 50% less than he earned on Sunday.
On Tuesday, he earned 20% more than he earned on Monday.
On Wednesday, he earned 30% less than he earned on Tuesday.

On Wednesday, he earned £84.

How much did Rob earn on Sunday?

$M = 0.5S$
$T = 1.2M$
$W = 0.7T$
$W = 84$
$S = ?$

A £15.12

B £35.28

C £117.60

D £200

E £210

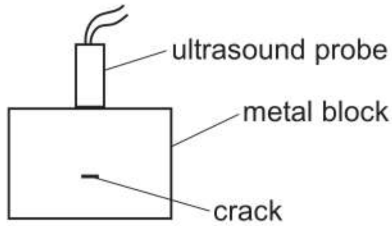
F £300

G £1200

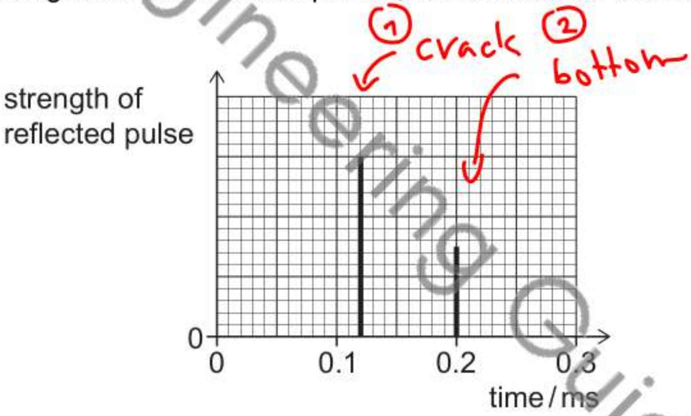
$$S = 2M = \frac{2}{1.2}T = \frac{2}{1.2 \times 0.7}W = \frac{1}{0.6 \times 0.7}W = \frac{W}{0.42}$$

$$= \frac{84}{0.42} = 200$$

6 Ultrasound is used to find a crack inside a cuboid block of metal. An ultrasound probe is held in contact with the top surface of the metal block and perpendicular to the surface. A short pulse of ultrasound is sent into the metal block at time $t = 0\,\text{ms}$ and reflects from both the crack and the bottom surface of the metal block.



The times between the emission of the ultrasound pulse and the return of the reflections to the probe, and the strengths of the reflected pulses, are measured. The results are shown on the graph.



The speed of ultrasound in the metal is $5000\,\text{m}\,\text{s}^{-1}$.

What is the distance between the **bottom surface** of the metal block and the crack?

A 0.2 m

B 0.3 m

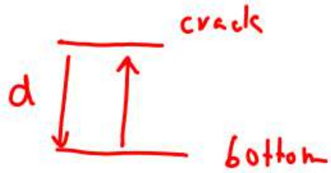
C 0.4 m

D 0.5 m

E 0.6 m

F 1.0 m

(2)



$\Delta t = 0.08\ \text{ms}$

$2d = v\,\Delta t$

$d = \frac{1}{2} v \Delta t$

$= \frac{1}{2} \times 5000 \times 0.08 \times 10^{-3}$

$= \frac{1}{2} \times 0.4 = 0.2\ \text{m}$

7 Which one of the following is a simplification of

$$\frac{5x^2-17x-12}{25x^2-9} \div \frac{x^2+x-12}{x^2-x-6} =$$

$$\frac{(5x+3)(x-4)}{(5x-3)(5x+3)} \cdot \frac{(x-3)(x+2)}{(x+4)(x-3)} =$$

$$\frac{(x-4)(x+2)}{(5x-3)(x+4)}$$

A $\frac{(x-4)(x+2)}{(x-3)(x+4)}$

B $\frac{(x-3)(x+2)}{(5x-3)(x+3)}$

C $\frac{(x-4)(x+2)}{(5x-3)(x+4)}$

D $\frac{(x-4)(x-3)}{(5x-3)(x-6)}$

E $\frac{(x+2)}{(5x+3)}$

F $\frac{(x+4)(x-6)}{(5x+3)(x+2)}$

G $\frac{(x-3)(x+2)}{(5x+3)(x+3)}$

8 Power is supplied to an electric motor at 0.800 kW.

The motor has an efficiency of 60% and is switched on for half an hour. $\eta = 0.6$

How much energy is **wasted** during this time? $E = ?$

A 0.160 J

B 0.240 J

C 160 J

D 240 J

E 576 J

F 864 J

G 576 000 J

H 864 000 J

$$E = (1-\eta)Pt$$
$$= 0.4 \times 0.8 \times 10^3 \times 30 \times 60$$
$$= 576 \times 10^3 \text{ J}$$

$$4 \times 8 \times 3 \times 6 = 2^6 \times 9$$
$$= 4^3 \times 9$$
$$= 576$$

$$\begin{array}{r} 64 \\ \times\ 9 \\ \hline 576 \end{array}$$

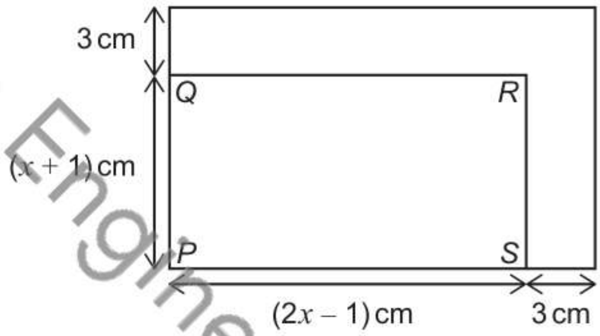
9 A rectangle *PQRS* has length $(2x - 1)$ cm and width $(x + 1)$ cm as shown on the diagram.

A larger rectangle is made by adding 3 cm to both the length and the width of *PQRS*, as shown.

The larger rectangle has an area of 360 cm^2.

$360 = (x+4)(2x+2)$

$\frac{x+1}{2x-1} = ?$



[diagram not to scale]

$360 = 2x^2 + 2x + 8x + 8$

$= 2x^2 + 10x + 8$

$0 = 2x^2 + 10x - 352$

$0 = x^2 + 5x - 176$

$x_{1/2} = \frac{-5 \pm \sqrt{25 + 4 \times 176}}{2}$

$x = 11$ cm

$\frac{x+1}{2x-1} = \frac{11+1}{22-1} = \frac{12}{21} = \frac{4}{7}$

What is the ratio of *PQ* to *PS*?

A 1 **:** 2

B 4 **:** 7

C 5 **:** 8

D 7 **:** 11

E 10 **:** 17

F 17 **:** 31

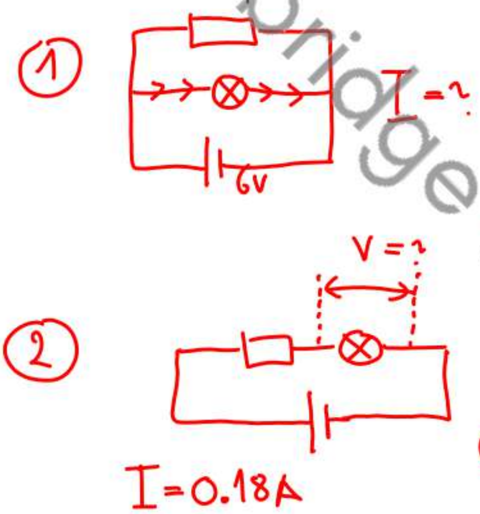
$\begin{array}{r} 176 \\ \times \quad 4 \\ \hline 704 \end{array}$

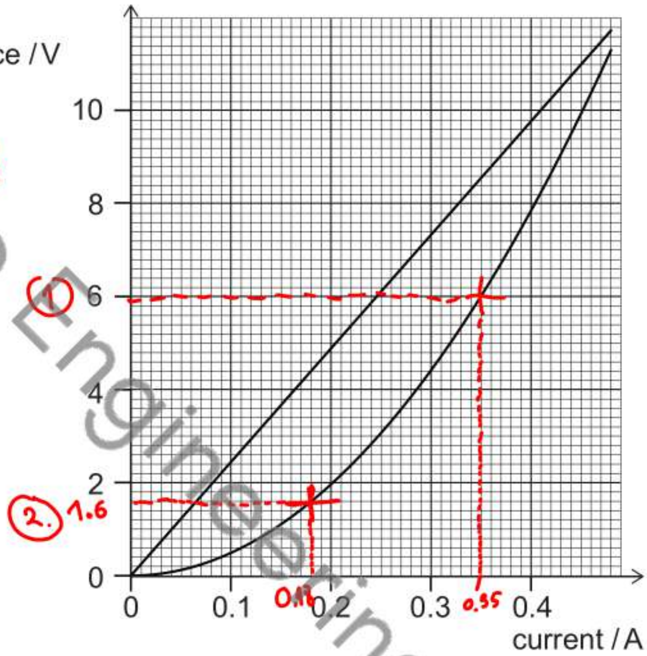
$\sqrt{25 + 704} =$

$\sqrt{729} =$

$\sqrt{9}\sqrt{81} = 3 \times 9 = 27$

10 The graph shows potential difference plotted against current for a filament lamp and a resistor.





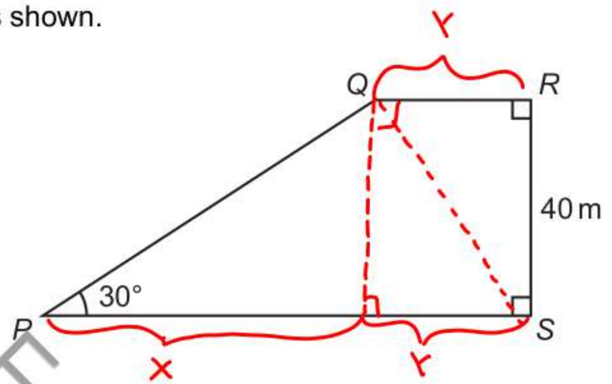
The lamp and the resistor are connected in parallel with each other to a 6.0 V power supply and the current in the lamp, I, is recorded.

In a second circuit, the lamp and the resistor are now connected in series with each other to the same power supply, and the current in the resistor is 0.18 A. The potential difference across the lamp, V, is recorded.

What are the values of I in the first circuit and V in the second circuit?

	I / A	V / V
A	0.25	1.6
B	0.25	3.0
C	0.25	4.4
D	0.35	1.6
E	0.35	3.0
F	0.35	4.4

11 *PQRS* is a trapezium as shown.



[diagram not to scale]

$\tan RSQ = \frac{5}{8}$

What is the length of *PS*, in metres?

A 45

B 65

C 80

D 120

E $25 + \frac{40\sqrt{3}}{3}$

F $40 + \frac{64\sqrt{3}}{3}$

G $25 + 40\sqrt{3}$

H $64 + 40\sqrt{3}$

$\tan(30^\circ) = \frac{40}{x} = \frac{1}{\sqrt{3}}$

$x = 40\sqrt{3}$ m //

$\tan(\angle RSQ) = \frac{5}{8} = \frac{Y}{40}$

$Y = 40 \times \frac{5}{8} = 25$ m //

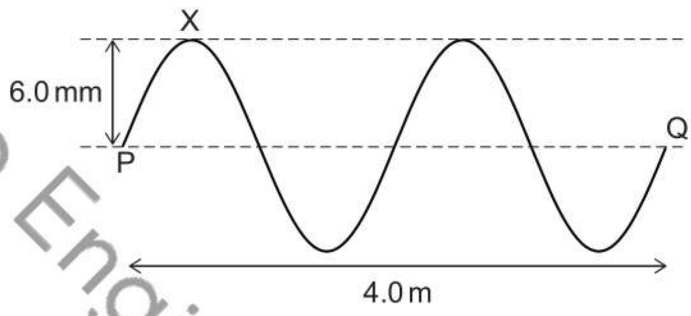
12 A transverse wave on a string has a speed of $500\,\mathrm{m\,s^{-1}}$.

The horizontal distance between two points P and Q on the wave is 4.0 m, as shown in the diagram.

$\lambda = 2\,\mathrm{m}$

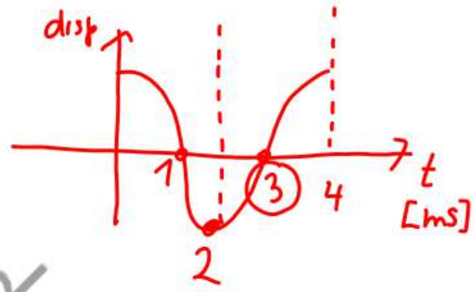
$T = \frac{\lambda}{c} = \frac{2}{500}$

$= 4\,\mathrm{ms}$



At time $t = 0$ ms, point X on the string is at its maximum displacement of 6.0 mm above equilibrium.

What is the displacement of point X at time $t = 7.0$ ms? $= 4 + 3$



A 6.0 mm above equilibrium

B between 0 mm and 6.0 mm above equilibrium

C 0 mm

D between 0 mm and 6.0 mm below equilibrium

E 6.0 mm below equilibrium

13 A solid cylinder has radius r cm and height h cm.

A cube has side length $3r$ cm.

The total surface area of the cylinder is equal to four times the total surface area of the cube.

Which of the following is an expression for h in terms of r?

A $\left(\dfrac{18}{\pi} - 2\right)r$

B $\left(\dfrac{18}{\pi} - 1\right)r$

C $\dfrac{27r}{\pi}$

D $\left(\dfrac{27}{\pi} - 1\right)r$

E $\left(\dfrac{27}{4\pi} - 1\right)r$

F $\dfrac{108r}{\pi}$

G $\left(\dfrac{108}{\pi} - 1\right)r$

H $\left(\dfrac{108}{\pi} - \dfrac{1}{2}\right)r$

$S_{\circ} = 4S_{\square}$

$S_{\circ} = 2\pi r^2 + 2\pi r h$

$S_{\square} = 6(3r)^2 = 54r^2$

$2\pi r^2 + 2\pi r h = 4 \times 54r^2$

$\pi r + \pi h = 108r$

$\pi h = r(108 - \pi)$

$h = r\left(\frac{108}{\pi} - 1\right)$

14 A piece of metal of mass 50 g is at thermal equilibrium in a hot liquid at temperature *T*.

The metal is removed from the liquid and immediately placed in 100 g of water that is at 20 °C.

The water is stirred and reaches a final temperature of 26 °C.

material	*specific heat capacity* / $J\,kg^{-1}\,°C^{-1}$
hot liquid	2000
metal	350
water	4200

What is the temperature *T* of the hot liquid? $T_f = 26°C$

(Assume that heat transfers to or from the surroundings are negligible.)

A 38 °C

B 51 °C

C 150 °C

D 170 °C

E 480 °C

$Q_M = Q_W$

$m_m c_M (T_0 - 26) = m_w c_w (26 - 20)$

$T_0 = 26 + \frac{m_w c_w}{m_m c_m} 6$ (change of units cancels out)

$= 26 + \frac{100 \times 4200}{50 \times 350} 6$

$= 26 + \frac{2 \times \cancel{70} \times 60}{\cancel{70} \times 5} 6 = 26 + 24 \times 6$

$= 26 + 144$

$= 170°C$

$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \end{array}$

15 The variables x and y are related by the equation:

$$x = 5 - \frac{2y^3 + 1}{1 - 2y^3}$$

Which of the following is a rearrangement to make y the subject?

A $y = \sqrt[3]{\dfrac{x-4}{8x-48}}$

B $y = \sqrt[3]{\dfrac{x-6}{8x-32}}$

C $y = \sqrt[3]{\dfrac{x-2}{x-6}}$

D $y = \sqrt[3]{\dfrac{x-3}{x-4}}$

E $y = \sqrt[3]{\dfrac{x-4}{2x-12}}$

F $y = \sqrt[3]{\dfrac{x-6}{2x-8}}$

$$5 - x = \frac{2y^3+1}{1-2y^3}$$

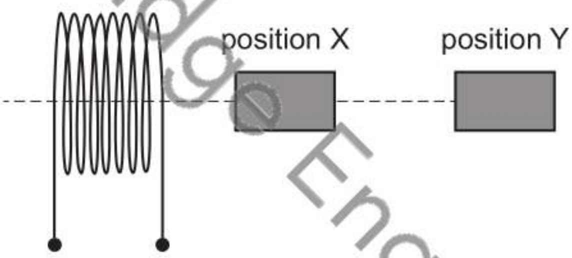
$$(5-x) - 2y^3(5-x) = 2y^3 + 1$$

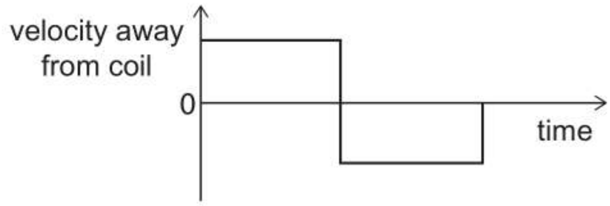
$$4 - x = y^3(12-2x)$$

$$y^3 = \frac{4-x}{12-2x}$$

16 A bar magnet is placed at position X close to one end of a coil and on the axis of the coil as shown.

The graph shows how the velocity of the magnet varies as it is then moved rapidly to position Y and back to position X.

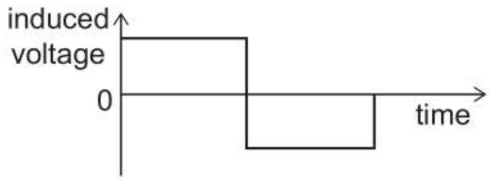




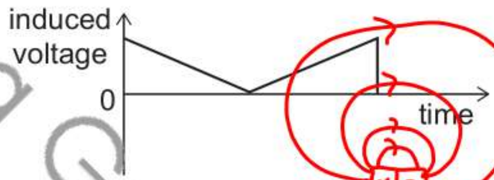
The magnetic field of the bar magnet still affects the coil when the magnet is at position Y.

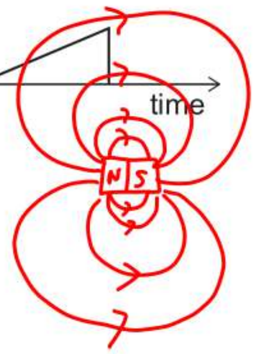
Which graph represents how the induced voltage in the coil changes as the magnet moves?

A



B





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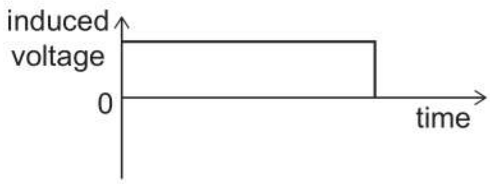
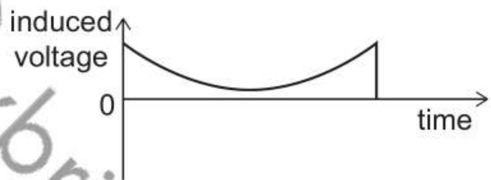


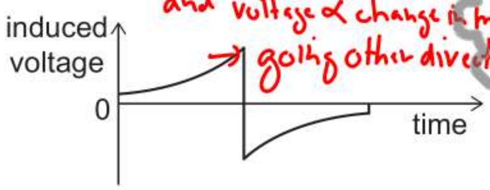
D

→ field stronger closer to the magnet
→ decrease nonlinear due to inhomogenous mag. field around magnet and voltage ∝ change in magnetic flux,
→ going other direction → symmetric result

E



F



17 Three different numbers are chosen at random from $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$.

What is the probability that the three numbers form the three sides of a right-angled triangle?

A $\frac{1}{15}$

B $\frac{1}{10}$

C $\frac{3}{10}$

D $\frac{1}{3}$

E $\frac{2}{5}$

F $\frac{2}{3}$

G $\frac{4}{5}$

$a^2 + b^2 = c^2$

$1+2=3$

$1+3=4$

$1+4=5$

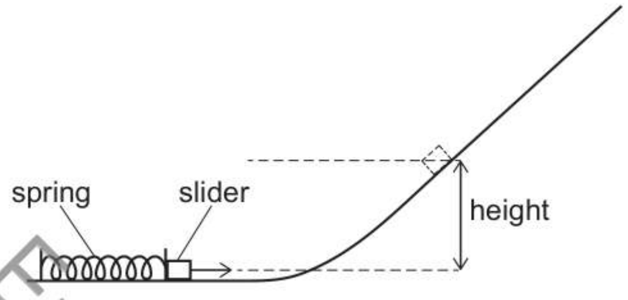
$2+3=5$

4

$\rightarrow$ number of conditions $= \binom{5}{3} = \frac{5!}{3!\,2!} = 10$

$p = \frac{4}{10} = \frac{2}{5}$

18 A small slider of mass 30 g is at rest near the bottom of a frictionless slope and in contact with a light uncompressed spring as shown.



[diagram not to scale]

The spring is compressed by 5.0 cm and the slider remains in contact with it.

The spring is released and causes the slider to rise up the slope to a maximum vertical height of 20 cm.

The slider is replaced with one of mass 20 g.

The spring is now compressed by 15 cm, and the new slider remains in contact with it.

To what maximum vertical height does this new slider rise after it is released?

(the spring obeys Hooke's law; assume that air resistance is negligible)

A 40 cm

B 60 cm

C 90 cm

D 120 cm

E 180 cm

F 270 cm

$\frac{1}{2}kx^2 = mgh$

constant from unit change

(1) $\frac{1}{2}k\,5^2 = 30\,g\,20 \times C$

(2) $\frac{1}{2}k\,15^2 = 20\,g\,H\,C$

divide (2) by (1)

$\frac{15^2}{5^2} = \frac{\cancel{20}H}{30 \times \cancel{20}}$ $\quad H = 30\,\frac{15^2}{5^2} = 30 \times 9 = 270\,\text{cm}$

19 The point $(-1, 5)$ is translated to the point $(3, 2)$ by two successive translations.

The first translation is by the vector $\begin{pmatrix} 3p \\ -4p \end{pmatrix}$

The second translation is by the vector $\begin{pmatrix} q \\ -2q \end{pmatrix}$

What is the value of $p + q$?

A -14

B -7

C -5

D -1

E 1

F 5

G 7

H 14

$$\begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 3p \\ -4p \end{pmatrix} + \begin{pmatrix} q \\ -2q \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$-1 + 3p + q = 3$ ① $3p + q = 4$

$5 - 4p - 2q = 2$ ② $+4p + 2q = +3$

subs ① from ② $4p + 2q - 3p - q = 3 - 4$

$p + q = -1$ //

20 A tall, smooth cylinder contains air at atmospheric pressure of 1.00×10^5 Pa. The density of the air in the cylinder is $1.20\,\text{kg m}^{-3}$.

A heavy piston is now placed in the top of the cylinder and allowed to fall slowly downwards, compressing the air until the piston rests in equilibrium.

The mass of the piston is 50.0 kg and its cross-sectional area is $0.0200\,\text{m}^2$.

What is the density of the air in the cylinder when the piston rests in equilibrium?

(gravitational field strength = $10\,\text{N kg}^{-1}$; assume that the air behaves as an ideal gas and that the temperature remains constant)

A $0.960\,\text{kg m}^{-3}$

B $1.20\,\text{kg m}^{-3}$

C $1.25\,\text{kg m}^{-3}$

D $1.28\,\text{kg m}^{-3}$

E $1.50\,\text{kg m}^{-3}$

F $4.80\,\text{kg m}^{-3}$

$pV = nRT$

$pV = \frac{m}{M} RT$

$pM = \rho RT$

$\frac{pT}{\rho} = \text{const}$

$p_0 = 1 \times 10^5$ Pa

$p_1 = p_0 + \frac{F}{S} = p_0 + \frac{mg}{S}$

$= 1 \times 10^5 + \frac{50 \times 10}{0.02}$

$= 1.25 \times 10^4$ Pa

$\frac{p_0}{\rho_0} = \frac{p_1}{\rho_1}$ $\quad \rho_1 = p_1 \frac{\rho_0}{p_0}$

$\rho_1 = 1.25\,\rho_0$

$= 1.25 \times 1.2$

$= 1.5$

$$\begin{array}{r} 1.25 \\ 1.20 \\ \hline 0 \\ 250 \\ 125 \\ \hline 1.5000 \end{array}$$

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PART B Advanced Mathematics and Advanced Physics

21 Find the value of

$$\int_1^4 \frac{2x^2-3}{x\sqrt{x}}\,dx$$

A $\frac{19}{3}$

B $\frac{37}{3}$

C $\frac{53}{3}$

D $\frac{73}{4}$

E $\frac{81}{4}$

F $\frac{87}{4}$

$$2\int_1^4 x^{0.5}\,dx - 3\int_1^4 x^{-1.5}\,dx =$$

$$2\left[\frac{2}{3}x^{1.5}\right]_1^4 - 3\left[-2x^{-0.5}\right]_1^4$$

$$\frac{4}{3}\left[x^{1.5}\right]_1^4 + 6\left[x^{-0.5}\right]_1^4$$

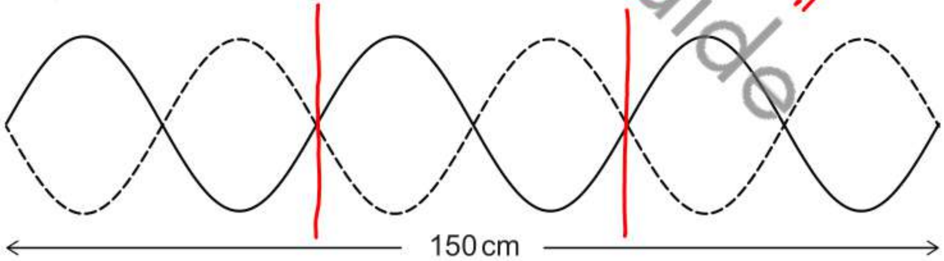
$$\frac{4}{3} * 7 + 6 * \left(-\frac{1}{2}\right) =$$

$$\frac{28-9}{3} = \frac{19}{3}$$

$4^{1.5} = 4\times\sqrt{4} = 8$

$4^{-0.5} = \frac{1}{2}$

22 The diagram represents a stationary wave in a medium.



The transverse waves that are creating the stationary wave travel at a speed of 300 m s^{-1} through the medium.

What is the frequency of the transverse waves?

A 75 Hz

B 150 Hz

C 200 Hz

D 450 Hz

E 600 Hz

F 1200 Hz

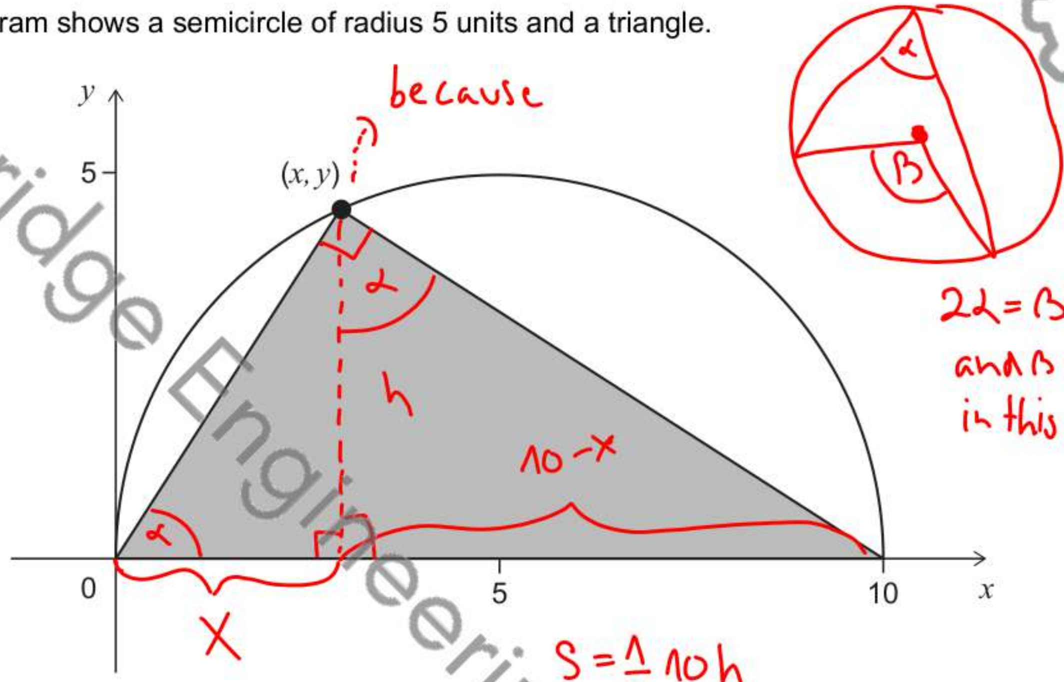
$\lambda = 50$ cm

$\lambda = cT$

$\lambda = \frac{c}{f}$

$$f = \frac{c}{\lambda} = \frac{300}{50\times10^{-2}} = 600\text{ Hz}$$

23 The diagram shows a semicircle of radius 5 units and a triangle.



$2\alpha = \beta$
and $\beta = 180°$
in this case

$S = \frac{1}{2} 10h$

The triangle has vertices at $(0, 0)$, $(10, 0)$ and (x, y).

$\triangle$ similarity: $\frac{x}{h} = \frac{h}{10-x}$

(x, y) is a point on the arc of the semicircle.

Which of the following is an expression in terms of x for the area of this triangle?

A $5\sqrt{10x - x^2}$

B $5x\sqrt{10 - x}$

C $5x\sqrt{10x - x^2 - 20}$

D $15x$

E $25x$

F $5x(10 - x)$

$h^2 = x(10-x)$
$h = \sqrt{x(10-x)}$

$S = 5h = 5\sqrt{x(10-x)}$

24 A stone is thrown vertically upwards from the surface of the Earth and reaches a maximum height h.

The same stone is thrown vertically upwards from the surface of the Moon, with the same initial speed.

What is the maximum height reached by the stone thrown on the Moon?

(gravitational field strength on the Earth = $10\,\mathrm{N\,kg^{-1}}$; gravitational field strength on the Moon = $1.6\,\mathrm{N\,kg^{-1}}$; air resistance may be ignored)

A $\left(\dfrac{10}{1.6}\right)h$

B $\left(\dfrac{10}{1.6}\right)^2 h$

C $\left(\dfrac{10}{3.2}\right)h$

D $\left(\dfrac{10}{3.2}\right)^2 h$

E $\left(\dfrac{10}{8.4}\right)h$

F $\left(\dfrac{10}{8.4}\right)^2 h$

G $\left(\dfrac{20}{1.6}\right)h$

H $\left(\dfrac{20}{1.6}\right)^2 h$

$E_E = E_M$

$m g_E h = m g_M h_M$

$h_M = \dfrac{g_E}{g_M} h = \dfrac{10}{1.6} h$ //

25 Four mathematically similar solids, W, X, Y and Z, have the following properties:

- The ratio of the lengths of W to the lengths of X is 1 **:** 2
- The ratio of the surface area of X to the surface area of Y is 2 **:** 1
- The ratio of the volume of Y to the volume of Z is 1 **:** 2 $Y < Z$

What is the order of the solids when arranged in **increasing** volume? → ordering by l identical

A W Y X Z

B W Y Z X

C W Z Y X

D Y W X Z

E Y W Z X

F Y Z W X

$S \propto l^2$

$V \propto l^3$

$$\frac{l_W}{l_X} = \frac{1}{2}$$

$$\frac{l_X^2}{l_Y^2} = \frac{2}{1} \qquad \frac{l_X}{l_Y} = \sqrt{2}$$

$$\frac{l_Y^3}{l_Z^3} = \frac{1}{2} \qquad \frac{l_Y}{l_Z} = \frac{1}{\sqrt[3]{2}}$$

$$l_W = \frac{1}{2} l_X$$

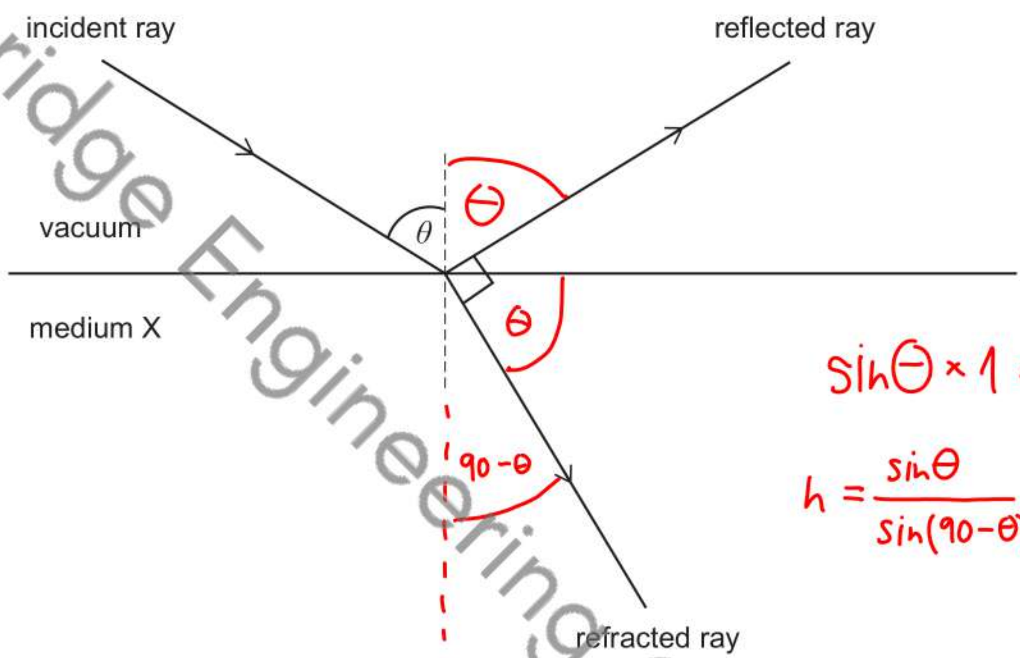
$$l_Y = \frac{1}{\sqrt{2}} l_X$$

$$l_Z = \frac{\sqrt[3]{2}}{2\sqrt{2}} l_X$$

$$\Rightarrow \quad \frac{1}{2} < \frac{1}{\sqrt{2}} < \frac{\sqrt[3]{2}}{2\sqrt{2}} < 1$$

$$W < Y < Z < X$$

26 A ray of light is incident on a boundary between a vacuum and medium X at an angle θ as shown:



$$\sin\theta \times 1 = \sin(90-\theta) \times n$$

$$n = \frac{\sin\theta}{\sin(90-\theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

The incident ray is partially reflected and partially refracted. The angle between the reflected and refracted rays is 90°.

What is the refractive index of medium X?

A $\sin\theta$

B $\dfrac{1}{\sin\theta}$

C $\cos\theta$

D $\dfrac{1}{\cos\theta}$

E $\tan\theta$

F $\dfrac{1}{\tan\theta}$

27 Which one of the following expressions is equal to

$$\sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$$

$$\frac{(2+\sqrt{20})^2}{(1+\sqrt{5})^3} = \frac{(2+2\sqrt{5})^2}{(1+\sqrt{5})^3} = \frac{4\cancel{(1+\sqrt{5})^2}}{(1+\sqrt{5})^{\cancel{3}\,1}}$$

$$= \frac{4}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}}$$

$$= \frac{4(1-\sqrt{5})}{1-5} = \sqrt{5}-1 \,//$$

A $\sqrt{5}-1$

B $\dfrac{\sqrt{5}-1}{2}$

C $\dfrac{6(5\sqrt{5}-1)}{31}$

D $\dfrac{3(5\sqrt{5}-1)}{31}$

E $\dfrac{-22+10\sqrt{2}+11\sqrt{5}-4\sqrt{10}}{2}$

F $\dfrac{-22+10\sqrt{2}+11\sqrt{5}-4\sqrt{10}}{4}$

28 An electric train is travelling along a straight horizontal track. It passes a point Q on the track at time $t = 0$.

The distance x that it then travels away from Q is given by the equation:

$$x = at + bt^2$$

b

$\boxed{\frac{1}{2} a t^2}$

where a and b are constants.

Which of the following statements is/are correct?

1 The speed of the train increases with time at a constant rate. ✓ since $x \propto t^2$

2 The resultant force acting on the train increases with time. → since $a = \text{const} \Rightarrow F = ma$, ✗ resultant $F = \text{const}$

3 The rate at which energy is transferred to the train increases with time. ✓ resitant F↑ = more power to sustain

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

29 Evaluate

$$B = \sum_{r=2}^{\infty}\left(\frac{1}{2^r}+\frac{1}{3^r}+\frac{1}{4^r}\right)$$

A $\frac{21}{40}$

B $\frac{3}{4}$

C $\frac{13}{12}$

D $\frac{157}{120}$

E $\frac{11}{6}$

F $\frac{23}{12}$

$$A = \sum_{r=0}^{\infty}\left(\frac{1}{2^r}+\frac{1}{3^r}+\frac{1}{4^r}\right) = \frac{1}{1-\frac{1}{2}} + \frac{1}{1-\frac{1}{3}} + \frac{1}{1-\frac{1}{4}}$$

$$= 2 + \frac{3}{2} + \frac{4}{3}$$

$$B = A - 3 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$$

$$= -1 + \frac{2}{2} + \frac{3}{3} - \frac{1}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

30 A length of wire has resistance R.

Another length of wire is made from the same material. This wire is twice as long as the first wire and has half the diameter.

Both wires have circular cross-sections.

The two wires are connected in parallel.

What is the total resistance of this combination?

$R \propto \frac{l}{d^2}$ $\quad A \propto d^2$

$R_1 = R \quad R_2 = 2\left(\frac{1}{\frac{1}{2}}\right)^2 R$

$= 8R$

A $\frac{2}{3}R$

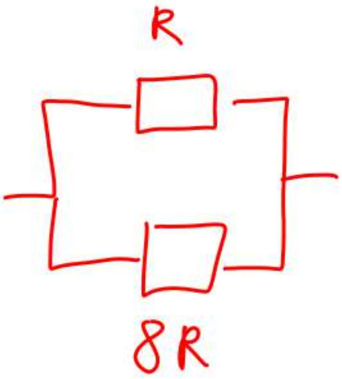
B $\frac{8}{9}R$

C $2R$

D $3R$

E $8R$

F $9R$



$$R_{NET} = \frac{R \times 8R}{R + 8R}$$

$$= \frac{8R^2}{9R} = \frac{8}{9}R \text{ //}$$

31 What is the complete set of real values of x for which

$$x^2\left(x^2+4\right) < 21$$

A $-\sqrt{3} < x < \sqrt{3}$

B $-\sqrt{7} < x < \sqrt{7}$

C $x < -\sqrt{3}$ or $x > \sqrt{3}$

D $x < -\sqrt{7}$ or $x > \sqrt{7}$

E $-\sqrt{7} < x < -\sqrt{3}$ or $\sqrt{3} < x < \sqrt{7}$

F $x < -\sqrt{7}$ or $-\sqrt{3} < x < \sqrt{3}$ or $x > \sqrt{7}$

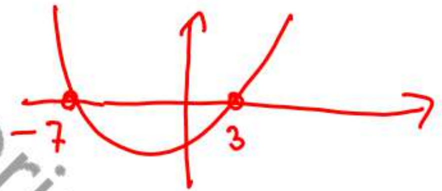
$x^2 = z$

$z(z+4) < 21$

$z^2 + 4z - 21 < 0$

$z_{1/2} = \dfrac{-4 \pm \sqrt{16-4(-21)}}{2} = -2 \pm 5 \;\begin{cases} 3 \\ -7 \end{cases}$

$z \in (-7, 3)$



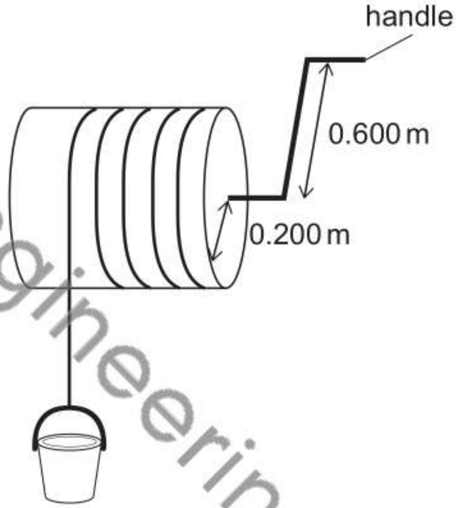
$z \in (-7, 3) \rightarrow x \in (?, ?)$

$z \in (0, 3) \rightarrow x \in (-\sqrt{3}, \sqrt{3})$

32 An empty bucket has a mass of 1.20 kg and an internal volume of 0.0150 m^3. The bucket is used to lift water from a well.

15 l ⇒ 15 kg

The bucket is attached to a light, inextensible rope which winds onto a rotating cylinder of radius 0.200 m when a handle is turned.



To lift a bucket completely full of water at constant speed, it is necessary to apply a force of 250 N to the handle that acts along the tangent to the circle of radius 0.600 m, in which the handle moves.

The energy required to lift the bucket is wasted energy.

What is the efficiency of the system in lifting water from the well?

(gravitational field strength = 10 N kg^{-1}; density of water = 1000 kg m^{-3})

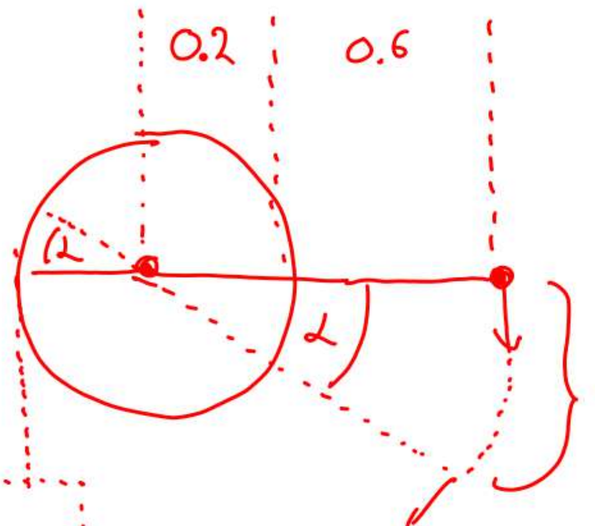
A 2.00%

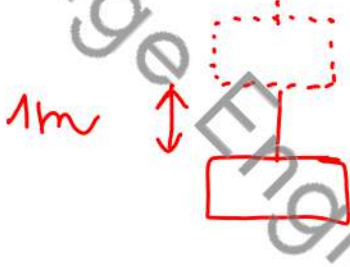
B 2.16%

C 20.0%

D 21.6%

E 60.0%





α in radians

$1 = 0.2\alpha$

$x = 0.6\alpha = \frac{0.6}{0.2} = 3\,m$

$E_{IN} = Fx = 250 \times 3 = 750\,J$

$E_{OUT} = mg\Delta h = 15 \times 10 \times 1 = 150\,J$

$\eta = \frac{E_{OUT}}{E_{IN}} = \frac{150}{750} = \frac{1}{5}$

α in radians

33 A sector of a circle has perimeter 24. $= \alpha r + 2r$

For what value of the radius does the sector have the maximum possible area?

A $3\sqrt{2}$

B $2\sqrt{6}$

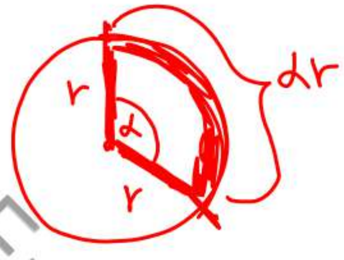
C $3\sqrt{6}$

D 6

E 12

F 18

G 36



$$24 = \alpha r + 2r$$

$$\alpha = \frac{1}{r}(24-2r)$$

$$A = \frac{\alpha}{2\pi}\pi r^2 = \frac{\alpha}{2}r^2 = \frac{1}{r}(24-2r)\frac{1}{2}r^2 = (12-r)r$$

$$\frac{dA}{dr} = -r + 12 - r = 0$$

$$r = 6$$

check whether maximum

$$\frac{d^2A}{dr^2} = -2 \checkmark$$

34 A uniform metre ruler has a mass of 100 g and is 1.00 m long.

A small object of mass 20 g is fixed at the 0 cm mark, and another small object of mass 80 g is fixed at the 100 cm mark.

The ruler is balanced on a pivot.

When the ruler is balanced, what distance is the pivot from the 0 cm mark?

A 15 cm

B 20 cm

C 25 cm

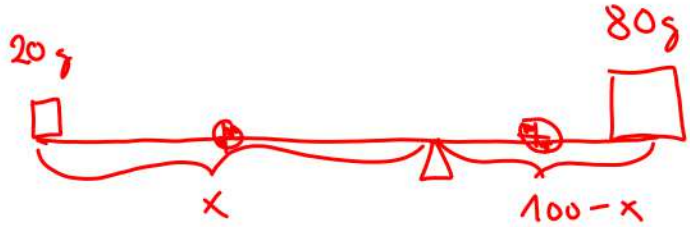
D 35 cm

E 65 cm

F 75 cm

G 80 cm

H 85 cm



$M_1 = M_2$ → all terms $\propto g$, so omitted

$$20 * x + \frac{x}{100}100 * \frac{1}{2}x = 80 * (100-x) + \frac{100-x}{100}100 * \frac{(100-x)}{2}$$

$$20x + \frac{1}{2}x^2 = 8000 - 80x + \frac{1}{2}\left((100)^2 - 200x + x^2\right)$$

$$200x = 8000 + 5000 = 13000$$

$$x = \frac{130}{2} = 65\,\text{cm}$$

35 The curve $y = x^2 - x - 6$ intersects the x-axis at the points A and B, and has a minimum at the point C.

The rectangle $ABDE$ has two of its vertices at A and B.

The point C lies on the edge DE, between D and E.

What is the area of the rectangle $ABDE$?

A 6

B 6.25

C 30

D 31.25

E 35

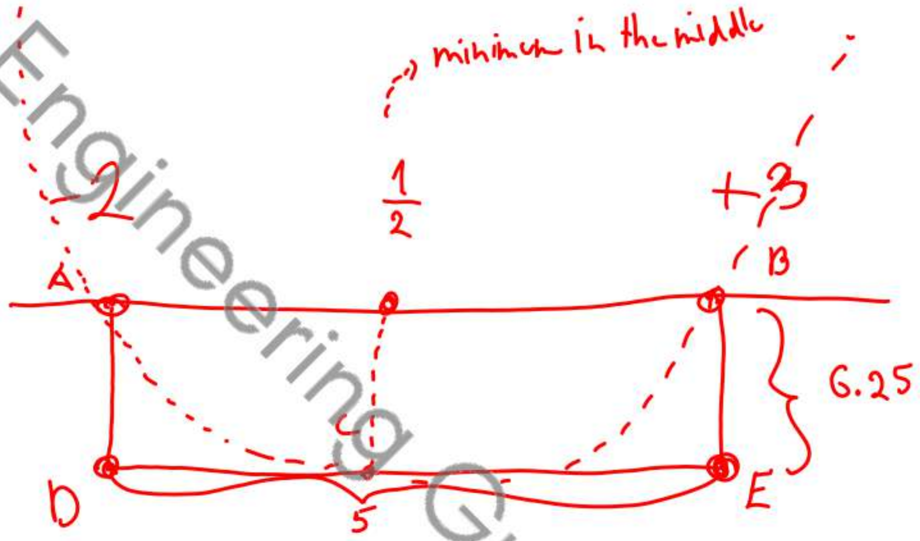
F 36.75

G 42

H 43.75

$x^2 - x - 6 = (x+2)(x-3)$

$y\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} - 6 = -6.25$



$S = 6.25 * 5 = 31.25$ //

36 A circuit comprising two resistors and two batteries, with negligible internal resistance, is set up as shown in the diagram. The two junctions in the circuit are labelled X and Y.

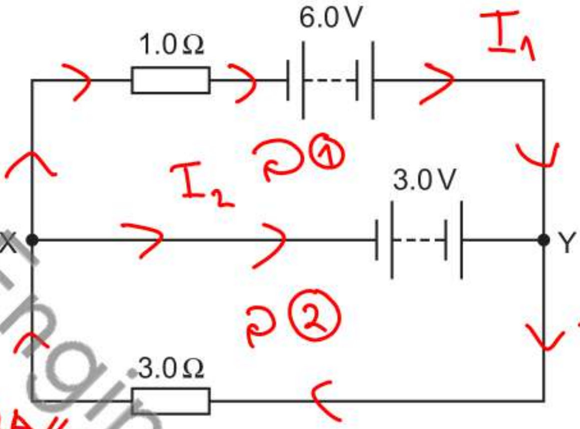
1. Guess current direction

2. Kirchhoff's voltage law

(1) $I_1 \times 1 - 6 + 3 = 0$

(2) $-3 + (I_1 + I_2) \times 3 = 0$

$\hookrightarrow I_1 = 3A \quad I_2 = 1 - I_1 = -2A$ //



What are the magnitude and direction of the current in the 3.0 V battery?

So current from Y → X

	magnitude of current / A	*direction of current*
A	1.0	from X to Y
B	2.0	from X to Y
C	2.25	from X to Y
D	6.0	from X to Y
E	1.0	from Y to X
F	2.0	from Y to X
G	2.25	from Y to X
H	6.0	from Y to X

37 A straight line passes through the points $(0, 2a)$ and $(a, 0)$, where a is a positive constant.

What is the perpendicular distance of the point $P\,(a, 2a)$ from this line?

A $\frac{\sqrt{2}}{5}a$

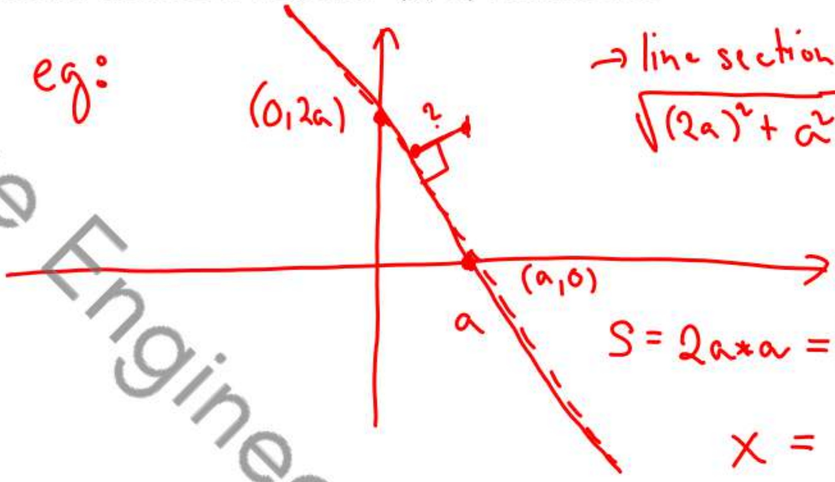
B $\frac{\sqrt{5}}{2}a$

C $\frac{2\sqrt{5}}{5}a$

D $\frac{4\sqrt{5}}{5}a$

E $\frac{3\sqrt{10}}{5}a$

eg:



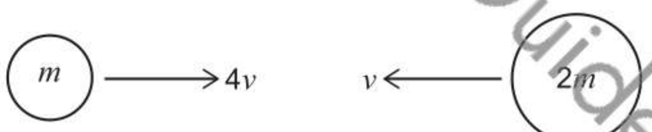
→ line section = $\sqrt{(2a)^2+a^2} = \sqrt{5}\,a$

Δ

$S = 2a*a = 2*\frac{1}{2}*X*\sqrt{5}a$

$X = \frac{2a}{\sqrt{5}} = \frac{2\sqrt{5}a}{5}$

38 Two isolated spheres have masses m and $2m$. They are moving towards each other along the same straight line with speeds $4v$ and v respectively as shown:



The spheres collide with each other and coalesce. → Completely inelastic collision

What is the loss of kinetic energy during the collision?

A $\frac{1}{3}mv^2$

B $\frac{2}{3}mv^2$

C $\frac{25}{3}mv^2$

D $\frac{26}{3}mv^2$

E $\frac{2}{9}mv^2$

F $\frac{79}{9}mv^2$

G $9mv^2$

→ momentum: $m4v - 2mv = 3mU$

$U = \frac{2}{3}V$

→ loss $= \frac{1}{2}m(4v)^2 + \frac{1}{2}2mv^2 - \frac{1}{2}3m\left(\frac{2}{3}v\right)^2$

$= \frac{1}{2}mv^2\left(16+2-\frac{4}{3}\right) = mv^2\left(9-\frac{2}{3}\right)$

$= mv^2\left(\frac{25}{3}\right)$ //

39 Find the sum of the real solutions to the equation

$$2^x - (\sqrt{2})^{x+6} + 12 = 0$$

$\sqrt{2}^x = z$

A 8

B 16

C $2^{\frac{4+\sqrt{2}}{2}}$

D $2^{\frac{6+\sqrt{2}}{2}}$

E $1 + \frac{1}{2}\log_2 3$

F $4 + 2\log_2 3$

$(\sqrt{2}^x)^2 - (\sqrt{2})^x * 2^3 + 12 = 0$

$z^2 - 8z + 12 = 0$

$(z-6)(z-2) = 0$

$(\sqrt{2})^{x_1} = 6 \Rightarrow x_1 \underbrace{\log_2(\sqrt{2})}_{\frac{1}{2}} = \underbrace{\log_2(6)}_{1+\log_2(3)}$

$x_1 = 2(1+\log_2 3)$

$(\sqrt{2})^{x_2} = 2 \Rightarrow x_2 = 2$

$x_1 + x_2 = 4 + 2\log_2 3$ //

40 A steel cable has mass 64 kg and cross-sectional area 2.0×10^{-4} m^2.

The Young modulus of steel is 2.0×10^{11} Pa.

When the cable lies on horizontal ground its length is 40 m.

What is its extension when it is suspended freely from one end and hangs vertically?

(gravitational field strength = 10 N kg^{-1}; assume that the cable obeys Hooke's law)

A 0 m

B 8.0×10^{-7} m

C 8.0×10^{-6} m

D 1.6×10^{-5} m

E 3.2×10^{-5} m

F 6.4×10^{-5} m

G 3.2×10^{-4} m

H 6.4×10^{-4} m

$p = k\varepsilon$ $\quad \Delta l = \int_0^{40} \varepsilon \, dl$

$\varepsilon = \frac{p}{k}$ $\quad \varepsilon(l) = \frac{\frac{l}{40} mg}{kA} = \frac{64 * 10}{40 * 2 * 10^{11} * 2 * 10^{-4}} l$

$= \frac{64 * 10}{16 * 10^8} = 4 * 10^{-7} l$

$\Delta l = 4 * 10^{-7} \int_0^{40} l \, dl$

$= 4 * 10^{-7} * \left[\frac{1}{2} l^2\right]_0^{40} = 4 * 10^{-7} * \frac{1}{2} 40^2$

$= 32 * 10^{-5}$ m

$= 3.2 * 10^{-4}$ m //

END OF TEST

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UNIVERSITY OF CAMBRIDGE

Engineering Admissions Assessment – Section 1 2022 D564/11

Candidate number: J

Centre number

Date of birth (DD MM YYYY)

First Name(s)

Surname / Family name

Fill in the appropriate circle for your chosen answer e.g.

A B C D E
○ ● ○ ○ ○

Use a soft pencil. If you make a mistake, erase thoroughly and try again.

1473984148

Part A: Maths & Physics

1	A B C D E F	11	A B C D E F G H
2	A B C D E F G H	12	A B C D E
3	A B C D E F G H	13	A B C D E F G H
4	A B C D E F G H	14	A B C D E
5	A B C D E F G	15	A B C D E F
6	A B C D E F	16	A B C D E F
7	A B C D E F G	17	A B C D E F G
8	A B C D E F G H	18	A B C D E F
9	A B C D E F	19	A B C D E F G H
10	A B C D E F	20	A B C D E F

Part B: Advanced Maths & Physics

21	A B C D E F	31	A B C D E F
22	A B C D E F	32	A B C D E
23	A B C D E F	33	A B C D E F G
24	A B C D E F G H	34	A B C D E F G H
25	A B C D E F	35	A B C D E F G H
26	A B C D E F	36	A B C D E F G H
27	A B C D E F	37	A B C D E
28	A B C D E F G H	38	A B C D E F G
29	A B C D E F	39	A B C D E F
30	A B C D E F	40	A B C D E F G H

38585

J

ENGINEERING
ADMISSIONS ASSESSMENT

D564/12

2022

60 minutes

SECTION 2

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 2 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 22 printed pages and 2 blank pages.

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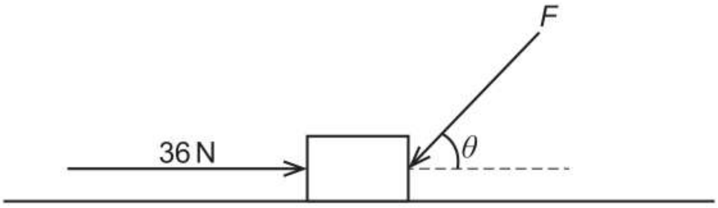
1 The diagram shows an object of mass 2.4 kg on a smooth horizontal surface.



A force F acts on the object at an acute angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

A force of 36 N acts on the object towards the right.

The object is in equilibrium.

What is the magnitude of the normal contact force exerted on the object by the surface?

(gravitational field strength = 10 N kg^{-1})

A 24 N

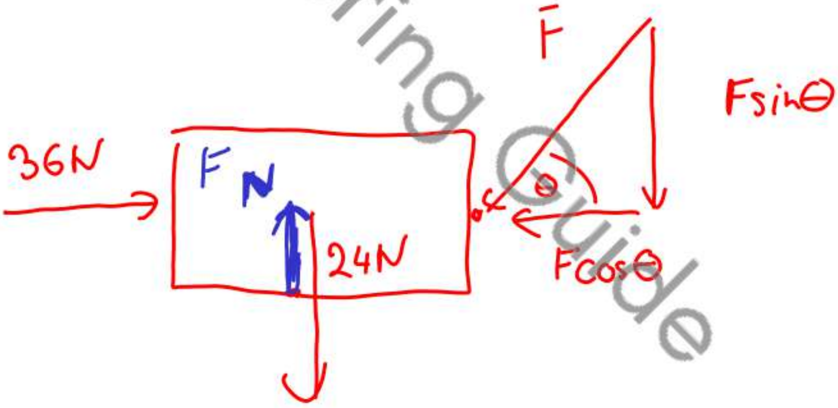
B 27 N

C 48 N

D 51 N

E 72 N

F 75 N



$$X: \quad 36 - F\cos\theta = 0 \qquad F = \frac{36}{\cos\theta}$$

$$Y: \quad 24 + F\sin\theta - F_N = 0$$

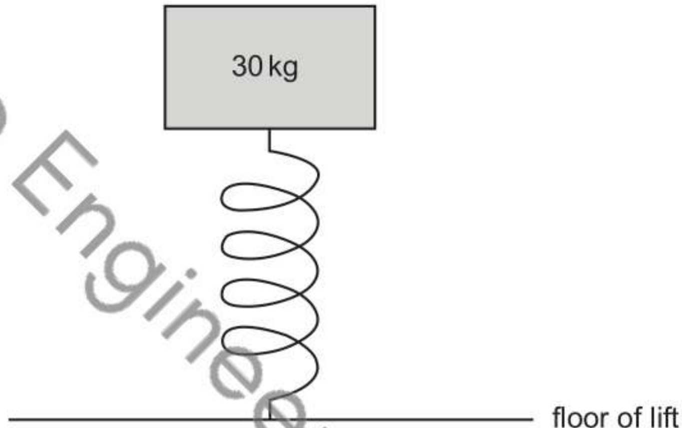
$$F_N = 24 + \frac{36}{\cos\theta}\sin\theta = 24 + 36 * \tan\theta$$

$$= 24 + \frac{4}{3} 36$$

$$= 72\,N$$

2 The length of a spring when no force acts on it is L. The spring constant of the spring is $3.0 \times 10^3\,\mathrm{N\,m^{-1}}$.

The spring is on the floor of an accelerating lift (elevator), and the spring supports a 30 kg mass.



The lift is accelerating downwards at $2.0\,\mathrm{m\,s^{-2}}$.

What is the difference between L and the length of the spring when the lift is accelerating downwards?

(gravitational field strength = $10\,\mathrm{N\,kg^{-1}}$; the spring obeys Hooke's law)

A 0 cm

B 2.0 cm

C 8.0 cm

D 10 cm

E 12 cm

$F = k\Delta x = m(g-a)$

$\Delta x = \dfrac{m(g-a)}{k} = \dfrac{30(8)}{3\times10^{3}} = 8\times10^{-2}\,m$ //

3 Electrical energy is transmitted at high voltage to a remote farm using an overhead power cable. Each of the two wires in the cable has a resistance of 2.5 Ω. The step-down transformer in the farm has a voltage ratio of 5.0. The transformer is ideal and 100% efficient. It supplies a power of 40 kW to a resistive load at the farm at a voltage of 250 V.

What is the rate at which electrical energy is transferred to thermal energy in the overhead cable?

A 1.28 kW

B 2.56 kW

C 5.12 kW

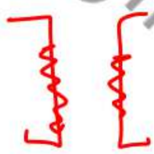
D 32 kW

E 64 kW

F 128 kW

$P = UI \qquad I = \frac{P}{U} = \frac{40 \times 10^3}{250} = \frac{4000 \times 10}{250}$

$= 160\,A$



$U_1 I_1 = U_2 I_2$

$I_1 = \frac{U_2}{U_1} I_2 = \frac{1}{5} 160 = 32\,A.$

$P = 2I^2R = 2 \cdot 32^2 \cdot 2.5 = 5 \cdot 1024$

$= 5.12 \times 10^3\,W$

$$\begin{array}{r} 32 \\ 32 \\ \hline 64 \\ 96 \\ \hline 1024 \end{array}$$

4 A wave is passing through a medium.

A particle of the medium has zero displacement from its equilibrium position at 0.12 s intervals, and at no other times.

The wavelength of the wave is greater than 10.0 m.

Two points are 5.0 m apart along the direction of travel of the wave.

The phase difference between the particles at the two points at the same instant is $\frac{\pi}{3}$ radians.

What is the speed of the wave?

A $1.8\,\mathrm{m\,s^{-1}}$

B $3.6\,\mathrm{m\,s^{-1}}$

C $7.2\,\mathrm{m\,s^{-1}}$

D $62.5\,\mathrm{m\,s^{-1}}$

E $125\,\mathrm{m\,s^{-1}}$

F $250\,\mathrm{m\,s^{-1}}$

$T = 2 \times 0.12 = 0.24\,s \qquad \lambda > 10\,m$

$c = \frac{\lambda}{T} > \frac{10}{0.24} \approx 40\,m\,s^{-1}$

$k \in \mathbb{Z}$

$\omega = \frac{2\pi}{T} \qquad \omega t = 2\pi k + \frac{\pi}{3}$

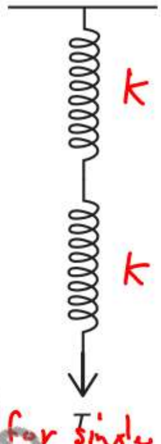
$\frac{\omega 5}{c} = 2\pi k + \frac{\pi}{3}$

$c = \frac{\frac{2\pi}{0.24} 5}{2\pi k + \frac{\pi}{3}} \approx \frac{40\pi}{2\pi k + \frac{\pi}{3}} = \frac{40}{2k + \frac{1}{3}}$

$k = 0 \quad c \approx \frac{40}{\frac{1}{3}} = 120\,m\,s^{-1}$ //

5 Three light springs, P, Q and R, are identical.

Springs P and Q are connected in series as shown. A downwards force T is applied to the lower end.



$k_{TOT} = \frac{1}{\frac{1}{k} + \frac{1}{k}} = \frac{k}{2}$ //

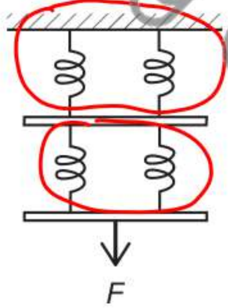
$k_{TOT} = k$

$k_{TOT} = \frac{1}{\frac{1}{k_i} + \frac{1}{k_i} + \frac{1}{k_i} + \frac{1}{k_i}} = \frac{k_i}{4}$

$k_i = 4k_{TOT}$

$4k$ for single

Spring R is cut into four equal lengths, and the four pieces arranged symmetrically as shown. The two connecting bars have negligible mass. A downwards force F is applied to the centre of the lower bar.



$4k + 4k = 8k$

$8k$

$k_{TOT} = \frac{1}{\frac{1}{8} + \frac{1}{8}} k = 4k$ //

The total extensions of the two systems are equal. The springs obey Hooke's law.

Which expression gives T in terms of F?

$\frac{F}{k_{TOT_2}} = \frac{T}{k_{TOT_1}}$

$T = \frac{k_{TOT1}}{k_{TOT2}} F = \frac{\frac{1}{2}}{4} F = \frac{1}{8} F$ ///

A $\frac{F}{16}$

B $\frac{F}{8}$

C $\frac{F}{4}$

D $\frac{F}{2}$

E $2F$

F $4F$

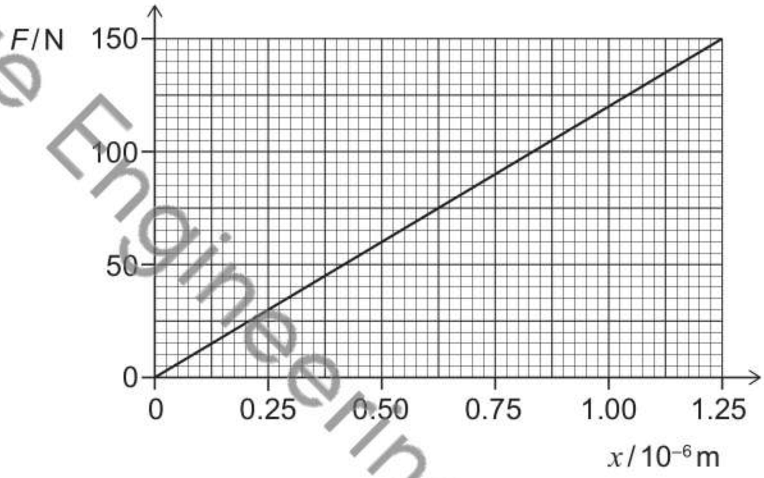
G $8F$

H $16F$

6 A nylon cube resting on a horizontal surface has a volume of $64\,\text{cm}^3$. $\Rightarrow$ 4cm side a

A force F is applied vertically downwards on the top face of the cube so that it compresses the height by x.

The graph shows the variation of F with x.



What is the Young modulus of the nylon?

(Assume that changes in horizontal cross-sectional area are negligible.)

A 7.7×10^3 Pa

B 4.8×10^6 Pa

C 9.6×10^6 Pa

D 1.2×10^8 Pa

E 1.5×10^9 Pa

F 3.0×10^9 Pa

G 1.9×10^{12} Pa

$$G = \frac{P}{\varepsilon} = \frac{Fa}{a^2 x} = \frac{F}{ax}$$

$$= \frac{150}{4 \times 10^{-2} \times 1.25 \times 10^{-6}}$$

$$= \frac{30}{10^{-8}} = 3 \times 10^9 \text{ Pa}$$

7 Five lampposts alongside a straight road are positioned at uniform intervals of 60 m.

A motorbike travelling at a constant velocity passes the first lamppost at time $t = 0$ s. It passes the fifth lamppost at $t = 20$ s.

$$\underbrace{\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet}_{240\,m}$$

A car travelling in the same direction as the motorbike is accelerating at $6.0\,m\,s^{-2}$. At time $t = 0$ s the car passes the first lamppost at a velocity of $3.0\,m\,s^{-1}$.

At what time t does the car overtake the motorbike?

A 1.5 s

B 2.0 s

C 2.5 s

D 3.0 s

E 3.5 s

F 4.0 s

G 5.0 s

$$V_M = \frac{\Delta s}{\Delta t} = \frac{240}{20} = 12\,m.s^{-1}$$

$$V_M t = V_c t + \frac{1}{2} a t^2$$

$$0 = (V_c - V_M)t + \frac{1}{2} a t^2$$

$$0 = t\underbrace{\left(V_c - V_M + \frac{1}{2} a t\right)}_{0}$$

$$t = \frac{2}{a}(V_M - V_c) = \frac{2}{6}(12 - 3) = 3\,s$$

8 An electrical appliance has an input power P which is a function of time t during the first 10 seconds after it is switched on.

This function is

$$P = 3t^2 + 4t$$

where P is in watts and t is in seconds.

The appliance is switched on at time $t = 0$.

The appliance has a constant efficiency of 90%.

What is the energy **wasted** by the appliance during the period $t = 2.0\,\text{s}$ to $t = 3.0\,\text{s}$ after it is switched on?

A 0.60 J

B 0.70 J

C 1.9 J

D 2.9 J

E 4.5 J

F 17 J

G 26 J

H 41 J

$$E_{TOT} = \int_2^3 (3t^2 + 4t)\,dt$$

$$= 3\left[\tfrac{1}{3}t^3\right]_2^3 + 4\left[\tfrac{1}{2}t^2\right]_2^3$$

$$= 3^3 - 2^3 + 2(3^2 - 2^2)$$

$$= 27 - 8 + 2(9-4) = 29\,\text{J}$$

$$E_{waste} = (1-\eta)E_{TOT} = 0.1 \times 29 = 2.9\,\text{J}$$

9 A solid cylinder is made of transparent glass of refractive index $\frac{2}{\sqrt{3}}$. It is surrounded by air.

A ray of light travelling in air hits the cylinder at the centre of one circular face at a non-zero angle θ to the normal, and refracts as it enters the cylinder.

The ray then strikes the curved surface of the cylinder at an angle of incidence equal to the critical angle.

What is the value of θ?

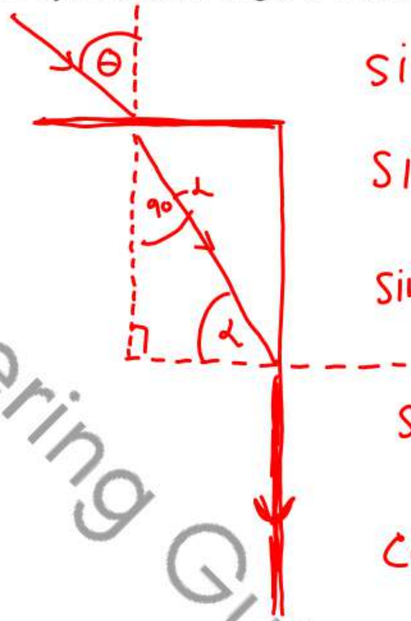
A $\sin^{-1}\frac{\sqrt{3}}{4}$

B $\sin^{-1}\frac{1}{\sqrt{3}}$

C $\sin^{-1}\frac{2}{\sqrt{6}}$

D $\sin^{-1}\frac{\sqrt{3}}{2}$

E $\sin^{-1}1$



$$\sin\alpha \frac{2}{\sqrt{3}} = 1$$

$$\sin\alpha = \frac{\sqrt{3}}{2}$$

$$\sin(90-\alpha) = \cos\alpha$$

$$\sin\theta = \cos\alpha \frac{2}{\sqrt{3}}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$= \sqrt{1-\frac{3}{4}} = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\cos\alpha \frac{2}{\sqrt{3}}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad //$$

10 An object of mass 20 kg is acted on by a force that varies in magnitude during the time interval $t = 0$ s to $t = 1.0$ s.

The force causes the object's displacement x to change with time t according to the relationship

$$x = -t^3 - 3t^2 + 4$$

where x is in metres and t is in seconds.

What is the magnitude of the $\underbrace{\text{impulse}}_{\text{momentum}}$ on the object over this time interval?

A 2.8 $kg\,m\,s^{-1}$

B 9.0 $kg\,m\,s^{-1}$

C 55 $kg\,m\,s^{-1}$

D 80 $kg\,m\,s^{-1}$

E 100 $kg\,m\,s^{-1}$

F 180 $kg\,m\,s^{-1}$

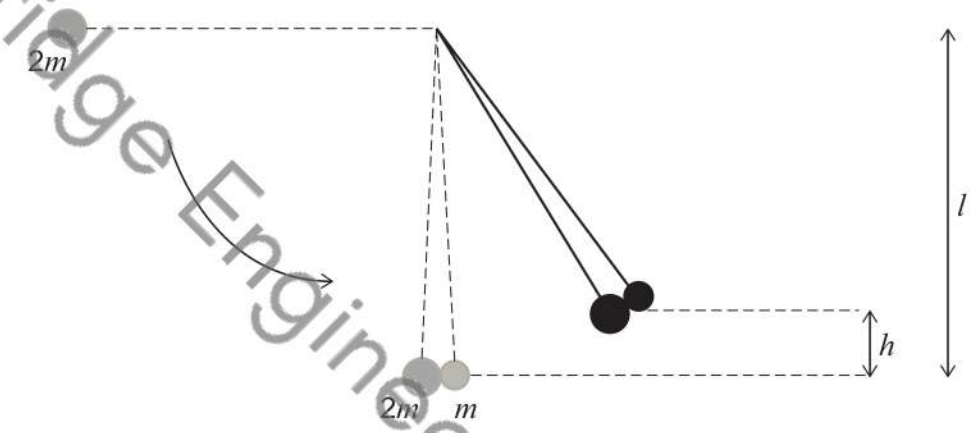
$\frac{dx}{dt} = v = -3t^2 - 6t$

$v(1) = -3 - 6 = -9\ m.s^{-1}$

$v(0) = 0\ m.s^{-1}$

$\Delta p = m\Delta v = 20 \times 9 = 180\ kg.m.s^{-1}$

11 Two small hard spheres of mass m and $2m$ are suspended side by side from light vertical strings of length l. The more massive sphere is raised so that its string is horizontal, and then released. It swings through 90° and strikes the smaller sphere. The two spheres stick together, and rise to a maximum height h as shown in the diagram.



[diagram not to scale]

Which expression gives the height h in terms of l?

1. inelastic collision
2. kin.en. → pot.en

(Assume that air resistance is negligible.)

A $\frac{4l}{27}$

B $\frac{8l}{27}$

C $\frac{4l}{9}$

D $\frac{2l}{3}$

E $\frac{8l}{9}$

F l

G $2l$

$$1.\ 2mU = 3mV$$

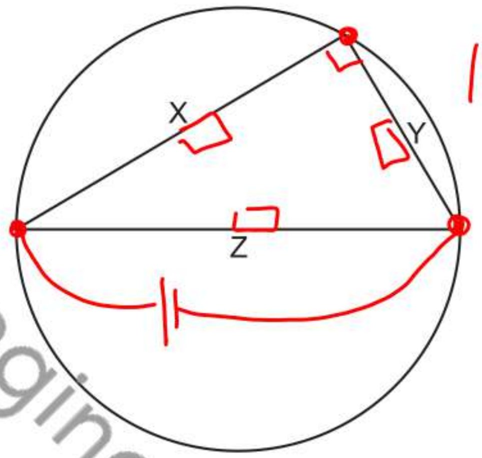
$$V = \sqrt{2gh} \quad U = \sqrt{2gl}$$

$$2\sqrt{2gl} = 3\sqrt{2gh}$$

$$2\sqrt{l} = 3\sqrt{h}$$

$$h = \left(\frac{2}{3}\right)^2 l = \frac{4}{9} l \ //$$

12 Three resistance wires X, Y and Z, made from the same metal, are connected to each other and to a circular plastic ring as shown.



$$|Y| = \sqrt{|Z|^2 - |X|^2} = \sqrt{15^2 - 12^2} = \sqrt{225 - 144} = \sqrt{81} = 9\,\text{cm}$$

[diagram not to scale]

Wires X and Y each have twice the diameter of wire Z.

Wire X is 12 cm long. Wire Z is 15 cm long and is connected across a diameter of the ring.

A power supply is connected to the two corners of the triangle that lie on the diameter.

What is the value of the ratio

$$\frac{\text{current in X}}{\text{current in Z}}\ ?$$

$$R \propto \frac{l}{d^2}$$

A $\frac{1}{5}$

B $\frac{7}{20}$

C $\frac{7}{10}$

D $\frac{5}{7}$

E $\frac{7}{5}$

F $\frac{10}{7}$

G $\frac{20}{7}$

H 5

$$R_X = \frac{1}{4} \cdot 12R \quad R_Y = \frac{1}{4} \cdot 9R \quad R_Z = 15R$$

$$I_X (R_X + R_Y) = I_Z R_Z$$

$$\frac{I_X}{I_Z} = \frac{R_Z}{R_X + R_Y} = \frac{15R}{3R + \frac{9}{4}R} = \frac{15}{\frac{21}{4}} = \frac{60}{21} = \frac{20}{7}$$

13 A light rope has cross-sectional area $6.0 \times 10^{-8}\,m^2$ and unstretched length 0.24 m.

The rope is fixed horizontally between two supports that are 0.24 m apart.

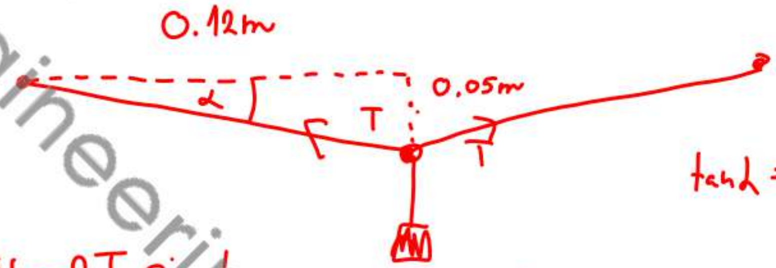
When a 1.0 kg mass is suspended from the middle of the rope, the vertical displacement of the middle of the rope from its original position is 0.050 m.

The rope obeys Hooke's law. Assume that changes in cross-sectional area are negligible.

What is the Young modulus of the material from which the rope is made?

(gravitational field strength = $10\,N\,kg^{-1}$)

144 − 25   119



A $5.2 \times 10^{8}\,N\,m^{-2}$

B $8.0 \times 10^{8}\,N\,m^{-2}$

C $1.0 \times 10^{9}\,N\,m^{-2}$

D $1.3 \times 10^{9}\,N\,m^{-2}$

E $2.0 \times 10^{9}\,N\,m^{-2}$

F $2.6 \times 10^{9}\,N\,m^{-2}$

G $5.2 \times 10^{9}\,N\,m^{-2}$

$\tan\alpha = \frac{5}{12}$

Y: $10N = 2T\sin\alpha$

$T = \frac{5}{\sin\alpha} = 13N$

$\Delta l = \frac{0.05}{\sin\alpha} - l = 0.01m$

$\tan\alpha = \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$

$\sin^2\alpha + \frac{\sin^2\alpha}{\tan^2\alpha} = 1$

$\sin\alpha = \frac{1}{\sqrt{1+\frac{1}{\tan^2\alpha}}} = \frac{1}{\sqrt{\frac{169}{25}}} = \frac{5}{13}$

$G = \frac{\sigma}{\varepsilon} = \frac{\frac{T}{S}}{\frac{\Delta l}{l}} = \frac{Tl}{S\Delta l}$

$G = \frac{13 \times 0.12}{6 \times 10^{-8} \times 0.01} = 13 \times 2 \times 10^{8}$

$= 2.6 \times 10^{9}\,N.m^{-2}$

14 The three graphs show the displacement, velocity and acceleration against time for an object moving in a straight line.

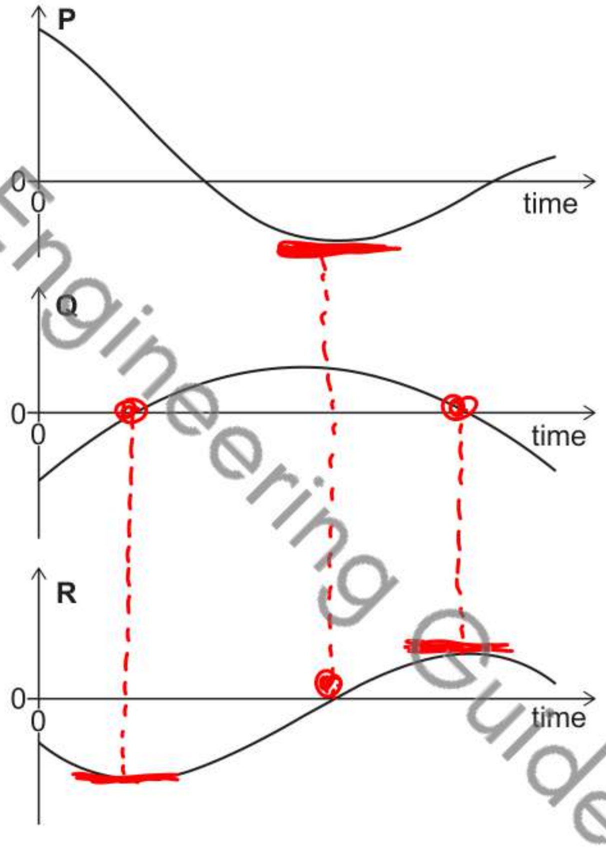
The time axis is shown to the same scale on all three graphs.

by observing where $\frac{d}{dt} = 0$:

$Q = \frac{dR}{dt}$

$R = \frac{dP}{dt}$

hence P = disp.
R = vel.
Q = acc.



Which graph represents which quantity?

	graph P	*graph Q*	*graph R*
A	acceleration	displacement	velocity
B	acceleration	velocity	displacement
C	displacement	acceleration	velocity
D	displacement	velocity	acceleration
E	velocity	acceleration	displacement
F	velocity	displacement	acceleration

15 A system of light springs that does not obey Hooke's law has an unstretched length of 2.0 m.

The extension x of the system is related to the force F applied to it by

$$F = px^2$$

where p is a constant.

A force of 2400 N increases the length of the system to 2.2 m.

How much work is done in increasing the length of the system from 3.0 m to 4.0 m?

A 1.2 kJ

B 60 kJ

C 70 kJ

D 120 kJ

E 140 kJ

F 740 kJ

$F = px^2 \quad p = \frac{F}{x^2} = \frac{2400}{(0.2)^2} = 60\,000 \text{ N.m}^{-2}$

$W = p \int_1^2 x^2 dx = p \left[\frac{1}{3}x^3\right]_1^2$

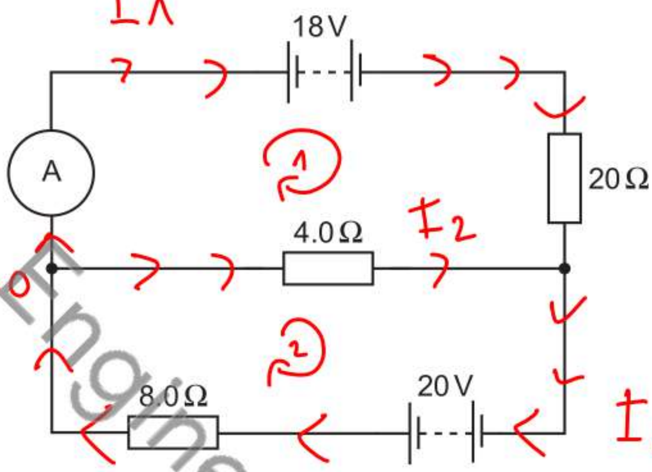
$= p\frac{1}{3}7 = \frac{7}{3}60\,000$

$= 140\,000 \text{ J}$ //

1st an 2nd Kirchhoff's law

16 The diagram shows a circuit that includes two batteries, each with negligible internal resistance.

I_1



I_1+I_2

1: $+18+20I_1-4I_2=0$

2: $4I_2-20+8(I_1+I_2)=0$

What is the reading on the ammeter?

A 0.0029 A

B 0.0071 A

C 0.063 A

D 0.083 A

E 0.50 A

F 0.65 A

G 1.2 A

H 2.0 A

A $10I_1-2I_2=-9$

B $8I_1+12I_2=20$

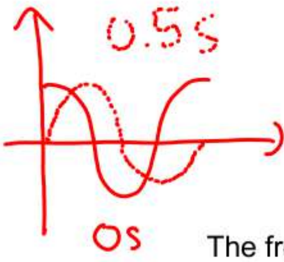
6A+B: $60I_1-12I_2+8I_1+12I_2=-54+20$

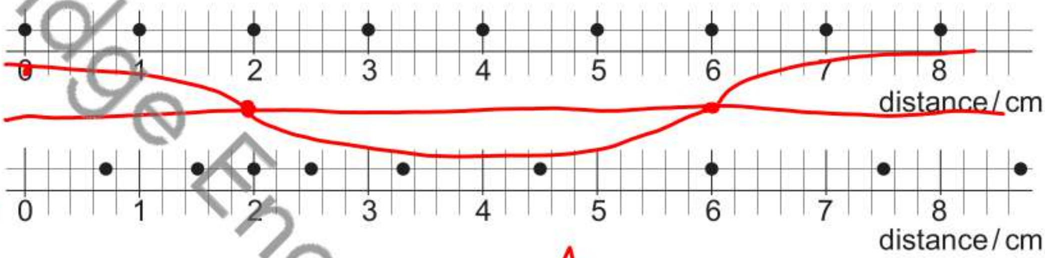
$68I_1=-34$

$I_1=-0.5A$ //

17 The upper diagram shows the equilibrium positions of nine equally spaced particles in a medium.

The lower diagram shows the positions of the same nine particles when a longitudinal wave is travelling through the medium. The wave is shown at time $t = 0$, travelling to the right.



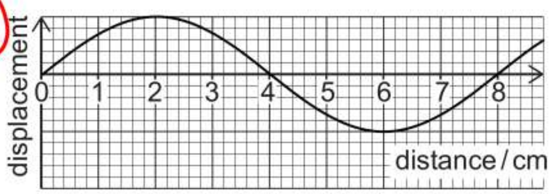


The frequency of the wave is 0.5 Hz. $T = \frac{1}{f} = 2s$ $\rightarrow \frac{1}{4}$ of period

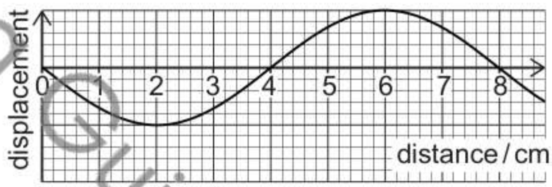
Which graph represents the displacements of the particles at a later time $t = 0.5$ s?

(On the graphs, positive displacement values represent particle displacements to the right.)

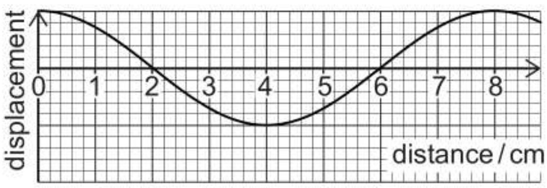
A



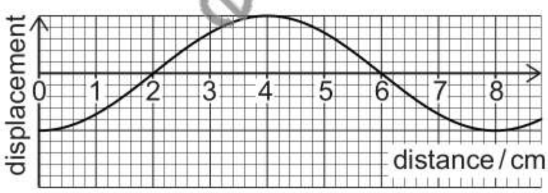
B



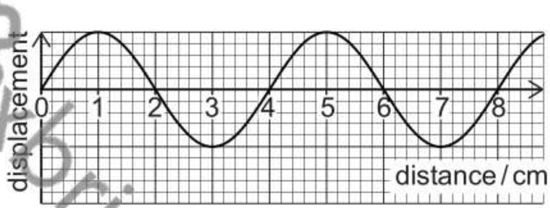
C



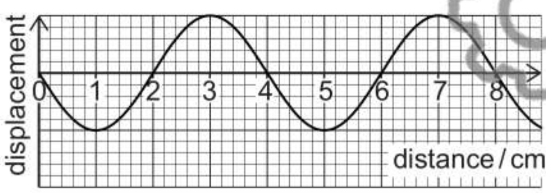
D



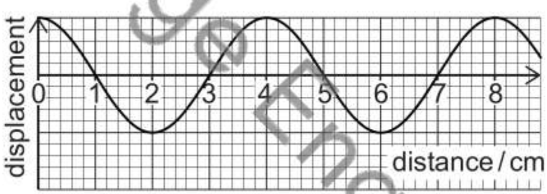
E



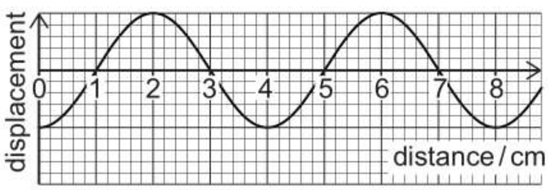
F



G



H



18 A power supply with constant emf and internal resistance r is connected to an external resistor.

The efficiency of the system is defined as

$$\text{efficiency} = \frac{\text{power dissipated by external resistor}}{\text{total power supplied by cell}}$$

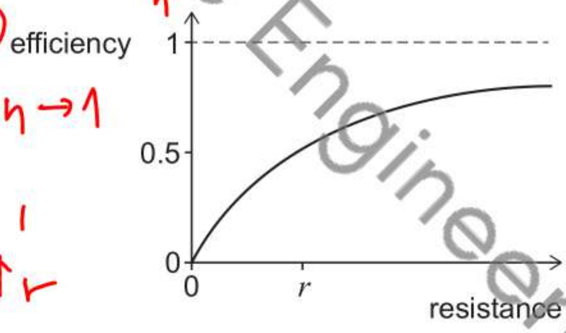
Which graph shows how the efficiency varies with the resistance of the external resistor?

η

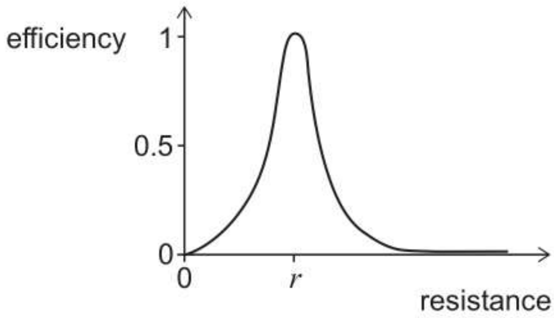
A

→ as $r \to \infty$, $\eta \to 1$

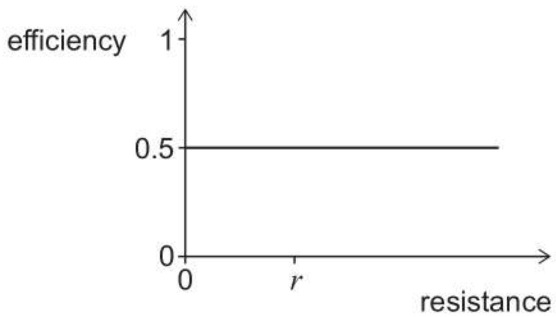
→ low η first, ↑η with ↑r



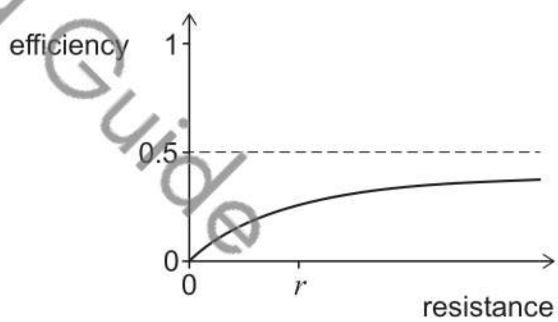
B



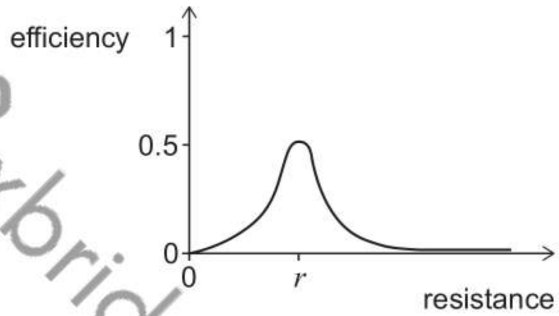
C



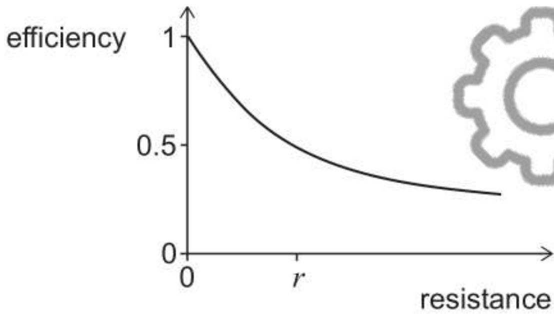
D



E



F



19 A 10 kg projectile is launched from ground level at an angle of 60° above the horizontal, with an initial speed of 12 m s^{-1}. The horizontal component of its velocity is to the right.

At the point during its flight when the vertical component of its velocity is zero, the projectile splits into two pieces, P and Q, each of mass 5 kg.

Immediately after the projectile splits, piece P has velocity 14 m s^{-1} to the right.

What is the speed of piece Q immediately before it hits the ground?

(Assume that air resistance is negligible, and that the ground is horizontal.)

A 2 m s^{-1}

B $\sqrt{31}$ m s^{-1}

C $6\sqrt{3}$ m s^{-1}

D $4\sqrt{7}$ m s^{-1}

E $2\sqrt{43}$ m s^{-1}

F $4\sqrt{13}$ m s^{-1}

G $4\sqrt{19}$ m s^{-1}

H $2\sqrt{127}$ m s^{-1}

→ cons. of momentum

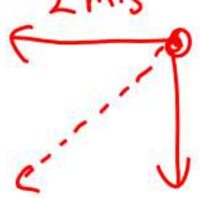
6 m.s^{-1} $V_2 = ?$ 14 m.s^{-1}

$2 \times 6 = 14 - V_2$

$V_2 = 2$ m.s^{-1}

$V_x = \cos 60 \, V = \frac{1}{2} \times 12 = 6$ m s^{-1}

Impact



$\sin 60^\circ \, V = \frac{\sqrt{3}}{2} 12 = 6\sqrt{3}$

$\sqrt{2^2 + 36 \times 3} = \sqrt{112} = 4\sqrt{7}$ m.s^{-1}

20 The density ρ of a sphere varies from its centre to its surface according to the equation

$$\rho = \rho_0\left(1 - \frac{x}{2R}\right)$$

where x is the distance from its centre, R is its radius and ρ_0 is the density at its centre.

What is the mass of the sphere?

(the surface area of a sphere of radius x is equal to $4\pi x^2$)

A $\dfrac{2\pi R^3\rho_0}{3}$

B $\dfrac{5\pi R^3\rho_0}{6}$

C $\dfrac{8\pi R^3\rho_0}{9}$

D $\pi R^3\rho_0$

E $\dfrac{29\pi R^3\rho_0}{24}$

F $\dfrac{19\pi R^3\rho_0}{15}$

G $\dfrac{4\pi R^3\rho_0}{3}$

H $2\pi R^3\rho_0$

$$dm = \rho\, dV = \rho\, 4\pi x^2 dx$$

$$m = \int dm = \int_0^R 4\pi \rho_0 \left(1 - \frac{x}{2R}\right) x^2\, dx$$

$$= 4\pi\rho_0\left(\left[\frac{1}{3}x^3\right]_0^R - \frac{1}{2R}\left[\frac{1}{4}x^4\right]_0^R\right)$$

$$= 4\pi\rho_0\left(\frac{1}{3}R^3 - \frac{1}{2R}\frac{1}{4}R^4\right)$$

$$= 4\pi\rho R^3 \underbrace{\left(\frac{1}{3} - \frac{1}{8}\right)}_{} = \frac{5}{6}\pi\rho R^3$$

END OF TEST

$$\frac{8-3}{24} = \frac{5}{24}$$

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UNIVERSITY OF CAMBRIDGE

Engineering Admissions Assessment – Section 2 2022 D564/12

Candidate number

J					

Centre number

Date of birth (DD MM YYYY)

First Name(s)

Surname / Family name

Fill in the appropriate circle for your chosen answer e.g.

A B C D E

○●○○○

Use a soft pencil. If you make a mistake, erase thoroughly and try again.

2881469910

1 A B C D E F ○○○○○○

2 A B C D E ○○○○○

3 A B C D E F ○○○○○○

4 A B C D E F ○○○○○○

5 A B C D E F G H ○○○○○○○○

6 A B C D E F G ○○○○○○○

7 A B C D E F G ○○○○○○○

8 A B C D E F G H ○○○○○○○○

9 A B C D E ○○○○○

10 A B C D E F ○○○○○○

11 A B C D E F G ○○○○○○○

12 A B C D E F G H ○○○○○○○○

13 A B C D E F G ○○○○○○○

14 A B C D E F ○○○○○○

15 A B C D E F ○○○○○○

16 A B C D E F G H ○○○○○○○○

17 A B C D E F G H ○○○○○○○○

18 A B C D E F ○○○○○○

19 A B C D E F G H ○○○○○○○○

20 A B C D E F G H ○○○○○○○○

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ENGAA 2022 Section 1 Answer Key

Question	Answer key
Q1	E
Q2	A
Q3	D
Q4	G
Q5	D
Q6	A
Q7	C
Q8	G
Q9	B
Q10	D
Q11	G
Q12	C
Q13	G
Q14	D
Q15	E
Q16	D
Q17	E
Q18	F
Q19	D
Q20	E
Q21	A
Q22	E
Q23	A
Q24	A
Q25	B
Q26	E
Q27	A
Q28	F
Q29	B
Q30	B
Q31	A
Q32	C
Q33	D
Q34	E
Q35	D
Q36	F
Q37	C
Q38	C
Q39	F
Q40	G

ENGAA 2022 Section 2 Answer Key

Question	Answer key
Q1	E
Q2	C
Q3	C
Q4	E
Q5	B
Q6	F
Q7	D
Q8	D
Q9	B
Q10	F
Q11	C
Q12	G
Q13	F
Q14	C
Q15	E
Q16	E
Q17	A
Q18	A
Q19	D
Q20	B